Chemistry 410A **Exam 1** **Fall 2011**

NAME:

Instructions:

1. Keep this exam closed until instructed to begin.
2. **Please write your name on this page but not on any other page.**
3. Please silence any noisy electronic devices you have.
4. Attached sheet(s) provide potentially useful constants and equations. You may detach these from the exam.
5. To receive full credit for your work, please
   (a) show all your work, using only the exam papers, including the back of this sheet if necessary;
   (b) specify the correct units, if any, for your final answers;
   (c) use an appropriate number of significant digits for final numerical answers;
   (d) **stop writing and close your exam immediately when time is called.**

Other notes:

- **The first page portion of the exam is worth 40 points.** Partial credit for these problems is not necessarily available.
- **Your 2 best scores of the 3 remaining problems will count towards the other 60 points.** Partial credit is available for these problems, so try each problem and do not erase any of your work.

1. **40 points.**

   (a) The boundary between the microwave and radio regions of the spectrum is at roughly $\lambda = 1.0\,\mathrm{m}$. What is the frequency of the radiation at this wavelength?

   (b) What is the energy in E_h (hartrees) of the $n = 4$ state of He^+?

   (c) The Bohr model of the atom successfully predicts which of the following (circle the number of each correct response):

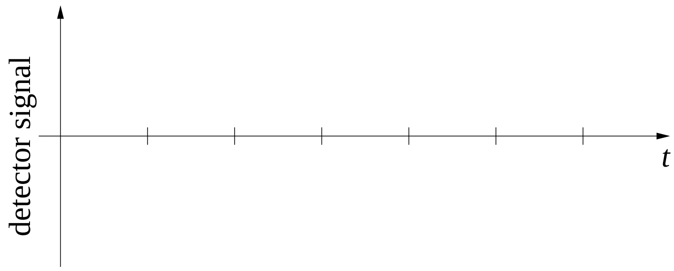
      i. The spectrum of the neutral helium atom.
      ii. The wavefunction of the electron in the neutral hydrogen atom.
      iii. The 2nd ionization energy of helium atom (the ionization energy of He^+).

   (d) A particle moves between two walls. The correspondence principle predicts that the system will behave more **classically** when we increase which of the following parameters (circle the number of each correct response):

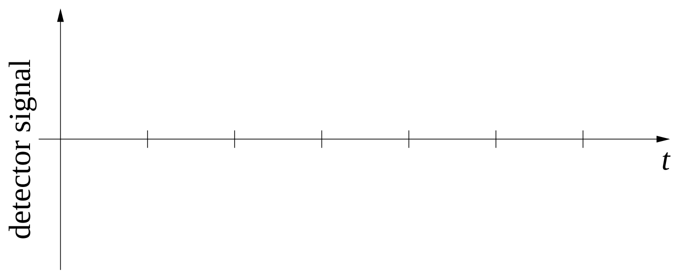
      i. The mass of the particle.
      ii. The spacing between the walls.
      iii. The de Broglie wavelength of the particle.
      iv. The speed of the particle.

2. A stationary detector directly measures the probability density of free particles called *neutrinos* as a function of time. We position the detector to monitor a beam of neutrinos at some constant speed v. Assume that the wave-like oscillations of the neutrinos all have the same phase, and that the neutrino mass is $m = 1.0 \cdot 10^{-37}\,\text{kg}$.

    (a) Sketch what the detector sees when $v_1 = 0.010c$, where c is the speed of light. Give numbers and units for the t axis, but not the vertical axis.



    (b) Do the same for $v_2 = 0.020c$, using the same scale for the t axis.



3. What is the eigenvalue when the $\hat{p}^2$ operator operates on the $n = 2$ state of a proton in a box of length $2.0\,\text{Å}$?

4. If the proton and electron in the Bohr model of hydrogen were replaced by uncharged particles with the same masses, there would still be a very small gravitational attraction between the particles. What is the ground state energy of this atom **in J**? The equation for the attractive force between two masses is

$$F_{\text{grav}} = \frac{m_1 m_2 G}{r^2},$$

where the gravitational constant G is equal to $6.67 \cdot 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$.

Balmer: $E_{\text{photon}} = (2.180 \cdot 10^{-18}\,\text{J}) \left(\frac{1}{\text{n}''^2} - \frac{1}{\text{n}'^2} \right)$

Bohr atom: $r_n = \frac{4\pi\epsilon_0 n^2 \hbar^2}{Z m_e e^2} = \frac{n^2}{Z} a_0 \qquad v_n = \frac{Ze^2}{4\pi\epsilon_0 n\hbar}$

$$E_n = -\frac{Z^2 m_e e^4}{(4\pi\epsilon_0)^2 2n^2\hbar^2} = -\frac{Z^2}{2n^2} E_{\text{h}} \qquad L_n = m_e r_n v_n = n\hbar$$

momentum operator: $\hat{p}_x = \frac{\hbar}{i}\frac{d}{dx}$

kinetic energy operator: $\hat{K} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

particle in a 1-D box: $\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \qquad E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$

Derivatives

$$\begin{aligned} dx^c &= cx^{c-1}dx\ (c \neq 0) & d(cx) &= cdx \\ d\ln x &= \tfrac{1}{x}dx & de^x &= e^x dx \\ d\sin x &= \cos x dx & d\cos x &= -\sin x dx \\ d[f(x)+g(x)] &= d[f(x)] + d[g(x)] & d[f(x)g(x)] &= f(x)d[g(x)] + g(x)d[f(x)] \end{aligned}$$

Integrals

$$\begin{aligned} \int x^n dx &= \tfrac{1}{n+1}x^{n+1} + C & \int a dx &= a(x + C) \\ \int \tfrac{1}{x}dx &= \ln x + C & \int e^x dx &= e^x + C \\ \int \ln x dx &= x\ln x - x + C & \int \tfrac{dx}{x(a+bx)} &= -\tfrac{1}{a}\ln\left(\tfrac{a+bx}{x}\right) + C \\ \int \sin x dx &= -\cos x + C & \int \cos x dx &= \sin x + C \\ \int \sin^2(ax)dx &= \tfrac{x}{2} - \tfrac{\sin(2ax)}{4a} + C & \int \cos^2(ax)dx &= \tfrac{x}{2} + \tfrac{\sin(2ax)}{4a} + C \\ \int [f(x)+g(x)]dx &= \int f(x)dx + \int g(x)dx & \int_a^b dx &= x|_a^b = b-a \\ \int_0^\infty x^n e^{-ax}dx &= \tfrac{n!}{a^{n+1}} & \int_0^\infty e^{-ax^2}dx &= \tfrac{1}{2}\left(\tfrac{\pi}{a}\right)^{1/2} \\ \int_0^\infty xe^{-ax^2}dx &= \tfrac{1}{2a} & \int_0^\infty x^2 e^{-ax^2}dx &= \tfrac{1}{4}\left(\tfrac{\pi}{a^3}\right)^{1/2} \\ \int_0^\infty x^{2n+1}e^{-ax^2}dx &= \tfrac{n!}{2a^{n+1}} & \int_0^\infty x^{2n}e^{-ax^2}dx &= \tfrac{[1\cdot3\cdot5\ldots(2n-1)]\sqrt{\pi}}{2^{n+1}a^{n+(1/2)}} \end{aligned}$$

Fundamental Constants

Avogadro's number	$\mathcal{N}_A$	$6.0221367 \cdot 10^{23}$ mol^{-1}
Bohr radius	$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$	$5.29177249 \cdot 10^{-11}$ m
Boltzmann constant	k_B	$1.380658 \cdot 10^{-23}$ J K^{-1}
electron rest mass	m_e	$9.1093897 \cdot 10^{-31}$ kg
fundamental charge	e	$1.6021773 \cdot 10^{-19}$ C
permittivity factor	$4\pi\epsilon_0$	$1.113 \cdot 10^{-10}$ C^2 J^{-1} m^{-1}
gas constant	R	8.314510 J K^{-1} mol^{-1}
	R	0.08314510 L bar K^{-1} mol^{-1}
	R	0.08206 L atm K^{-1} mol^{-1}
hartree	$E_\mathrm{h} = \frac{m_e e^4}{(4\pi\epsilon_0)^2\hbar^2}$	$4.35980 \cdot 10^{-18}$ J
Planck's constant	h	$6.6260755 \cdot 10^{-34}$ J s
	$\hbar$	$1.05457266 \cdot 10^{-34}$ J s
proton rest mass	m_p	$1.6726231 \cdot 10^{-27}$ kg
neutron rest mass	m_n	$1.6749286 \cdot 10^{-27}$ kg
speed of light	c	$2.99792458 \cdot 10^8$ m s^{-1}

Unit Conversions

	K	cm^{-1}	kJ mol^{-1}	kcal mol^{-1}	erg	kJ
kHz =	$4.799 \cdot 10^{-8}$	$3.336 \cdot 10^{-8}$	$3.990 \cdot 10^{-10}$	$9.537 \cdot 10^{-11}$	$6.626 \cdot 10^{-24}$	$6.626 \cdot 10^{-34}$
MHz =	$4.799 \cdot 10^{-5}$	$3.336 \cdot 10^{-5}$	$3.990 \cdot 10^{-7}$	$9.537 \cdot 10^{-8}$	$6.626 \cdot 10^{-21}$	$6.626 \cdot 10^{-31}$
GHz =	$4.799 \cdot 10^{-2}$	$3.336 \cdot 10^{-2}$	$3.990 \cdot 10^{-4}$	$9.537 \cdot 10^{-5}$	$6.626 \cdot 10^{-18}$	$6.626 \cdot 10^{-28}$
K =	1	0.6950	$8.314 \cdot 10^{-3}$	$1.987 \cdot 10^{-3}$	$1.381 \cdot 10^{-16}$	$1.381 \cdot 10^{-26}$
cm^{-1} =	1.4388	1	$1.196 \cdot 10^{-2}$	$2.859 \cdot 10^{-3}$	$1.986 \cdot 10^{-16}$	$1.986 \cdot 10^{-26}$
kJ mol^{-1} =	$1.203 \cdot 10^2$	83.59	1	0.2390	$1.661 \cdot 10^{-14}$	$1.661 \cdot 10^{-24}$
kcal mol^{-1} =	$5.032 \cdot 10^2$	$3.498 \cdot 10^2$	4.184	1	$6.948 \cdot 10^{-14}$	$6.948 \cdot 10^{-24}$
eV =	$1.160 \cdot 10^4$	$8.066 \cdot 10^3$	96.49	23.06	$1.602 \cdot 10^{-12}$	$1.602 \cdot 10^{-22}$
hartree =	$3.158 \cdot 10^5$	$2.195 \cdot 10^5$	$2.625 \cdot 10^3$	$6.275 \cdot 10^2$	$4.360 \cdot 10^{-11}$	$4.360 \cdot 10^{-21}$
erg =	$7.243 \cdot 10^{15}$	$5.034 \cdot 10^{15}$	$6.022 \cdot 10^{13}$	$1.439 \cdot 10^{13}$	1	10^{-10}
J =	$7.243 \cdot 10^{22}$	$5.034 \cdot 10^{22}$	$6.022 \cdot 10^{20}$	$1.439 \cdot 10^{20}$	10^7	10^{-3}
dm^3 bar =	$7.243 \cdot 10^{24}$	$5.034 \cdot 10^{24}$	$6.022 \cdot 10^{22}$	$1.439 \cdot 10^{22}$	$1.000 \cdot 10^9$	0.1000
kJ =	$7.243 \cdot 10^{25}$	$5.034 \cdot 10^{25}$	$6.022 \cdot 10^{23}$	$1.439 \cdot 10^{23}$	10^{10}	1

distance	1 Å =	10^{-10} m	
mass	1 amu =	$1.66054 \cdot 10^{-27}$ kg	
energy	1 J =	1 kg m^2 s^{-2}	= 10^7 erg
force	1 N =	1 kg m s^{-2}	= 10^5 dyn
electrostatic charge	1 C =	1 A s	= $2.9979 \cdot 10^9$ esu
	1 D =	$3.3357 \cdot 10^{-30}$ C m	= $1 \cdot 10^{-18}$ esu cm
magnetic field strength	1 T =	1 kg s^{-2} A^{-1}	= 10^4 gauss
pressure	1 Pa =	1 N m^{-2}	= 1 kg m^{-1} s^{-2}
	1 bar =	10^5 Pa	= 0.98692 atm