Chemistry 410A **Exam 1** **Fall 2008**

Solutions

1. (a) How much time does it take a photon to travel 20.0 m through a vacuum? **Solution:**

$$t = \frac{\text{distance}}{\text{speed}} = \frac{20.0\,\text{m}}{2.998 \cdot 10^{8}\,\text{m}\,\text{s}^{-1}} = \boxed{6.67 \cdot 10^{-8}\,\text{s}}.$$

(b) Which of the following changes decrease λ_{dB} of a hydrogen atom traveling in a 1-D box? **Solution:** Using $\lambda_{\text{dB}} = h/p$, we can ask, what is the effect of each change on the momentum? If p increases, then λ_{dB} decreases:

i. Replacing the hydrogen atom with a helium atom, keeping E the same. $\boxed{\text{yes}}$, higher mass, higher p.

ii. Doubling the speed of the atom. $\boxed{\text{yes}}$, higher speed, higher p.

iii. Decreasing the particle-in-a-box quantum number n by 1. $\boxed{\text{no}}$, lower energy, lower p.

iv. Reducing the length of the container by half, keeping n the same. $\boxed{\text{yes}}$, to keep n constant in smaller box need to raise E, so higher p.

(c) Calculate the energy in J of the $n = 2$ state of a proton in a one-dimensional box of length 4.0 Å. **Solution:**

$$E_2 = \frac{n^2\pi^2\hbar^2}{2ma^2} = \frac{2^2\pi^2(1.055 \cdot 10^{-34}\,\text{J}\,\text{s})^2}{2(1.673 \cdot 10^{-27}\,\text{kg})(4.0\,\text{Å})^2} = \boxed{8.2 \cdot 10^{-22}\,\text{J}}.$$

(d) If d/dx operates on $f(x) = 2e^{-3x}$, what is the eigenvalue? **Solution:**

$$\frac{d}{dx}f(x) = (-3)f(x) \qquad \text{eigenvalue} = -3$$

2. If the uncertainty in position of an electron is $\delta x = 1.0$ Å and its average speed is $3.0 \cdot 10^{6}\,\text{m}\,\text{s}^{-1}$, find the *minimum uncertainty* in that electron's de Broglie wavelength. The relationship between the uncertainties is given by $\delta p/\delta\lambda_{\text{dB}} = |dp/d\lambda_{\text{dB}}|$. **Solution:** For the *minimum* uncertainty, use the "=" sign:

$$\delta p = \frac{\hbar}{2\,\delta x}$$

$$\lambda_{\text{dB}} = \frac{h}{p} = \frac{6.626 \cdot 10^{-34}\,\text{J}\,\text{s}}{(9.109 \cdot 10^{-31}\,\text{kg})(3.0 \cdot 10^{6}\,\text{m}\,\text{s}^{-1})} = \boxed{2.4 \cdot 10^{-10}\,\text{m}}$$

$$\frac{d\lambda_{\text{dB}}}{dp} = -\frac{h}{p^2}$$

$$\frac{\delta\lambda_{\text{dB}}}{\delta p} = \frac{h}{p^2}$$

$$\delta\lambda_{\text{dB}} = \frac{h}{p^2}\,\delta p = \frac{h}{p^2}\frac{\hbar}{2\,\delta x}$$

$$= \frac{\lambda_{\text{dB}}^2}{4\pi\,\delta x} = \frac{(2.4 \cdot 10^{-10}\,\text{m})^2}{4\pi(1.0 \cdot 10^{-10}\,\text{m})} = \boxed{4.6 \cdot 10^{-11}\,\text{m}}.$$

3. The photon that excites the $n = 1 \to 10$ in He^+ has the same energy necessary to excite the $n = 5 \to 6$ transition in what other one-electron ion? **Solution:** Call the unknown atomic number Z and set the two transition energies equal:

$$\begin{aligned}
\Delta E_{\mathrm{He}^+} &= -\frac{2^2}{2}\left(\frac{1}{10^2} - \frac{1}{1^2}\right) E_{\mathrm{h}} = 1.98\, E_{\mathrm{h}} \\
&= -\frac{Z^2}{2}\left(\frac{1}{6^2} - \frac{1}{5^2}\right) E_{\mathrm{h}} = 0.006111\, Z^2\, E_{\mathrm{h}} \\
Z^2 &= \frac{1.98}{0.006111} = 324 \\
Z &= 18
\end{aligned}$$

The one-electron atom is $\boxed{\mathrm{Ar}^{17+}}$.

4. Find an equation for the root mean square speed $\langle v^2 \rangle^{1/2}$ of a particle with mass m in a one-dimensional box of length a and quantum state n. **Solution:** The energy of the particle in a one-dimensional box is all kinetic energy, which we can set equal to $mv^2/2$. From this we can solve for v^2 and take the square root to get $\langle v^2 \rangle^{1/2}$:

$$\begin{aligned}
E &= \frac{n^2 \pi^2 \hbar^2}{2ma^2} = \frac{mv^2}{2} \\
v^2 &= \frac{2E}{m} = \frac{n^2 \pi^2 \hbar^2}{m^2 a^2} \\
\langle v^2 \rangle^{1/2} &= \boxed{\frac{n\pi\hbar}{ma}}.
\end{aligned}$$