Chemistry 410A **Exam 1** **Spring 2016**

Solutions

1. **40 points.**

   (a) The highest measured energy for a photon is (at this writing) 16 TeV, where 1 TeV $= 1.602 \cdot 10^{-6}$ J. Calculate the wavelength in meters for this photon. **Solution:**

$$\lambda = \frac{hc}{E_{\text{photon}}} = \frac{(6.626 \cdot 10^{-34}\,\text{J s})(2.998 \cdot 10^{8}\,\text{m s}^{-1})}{(16\,\text{TeV})(1.602 \cdot 10^{-6}\,\text{J/TeV})} = \boxed{7.8 \cdot 10^{-21}\,\text{m.}}$$

   (b) Calculate the transition energy from the (2,2,2) to the (3,3,3) state of an electron in a cube-shaped box if the energy of the (1,1,1) state is $1.0 \cdot 10^{-17}$ J. **Solution:**

$$\begin{aligned} E &= \varepsilon_0(n_x^2 + n_y^2 + n_z^2) \\ E_{1,1,1} &= \varepsilon_0(1^2 + 1^2 + 1^2) = 3\varepsilon_0 = 1.0 \cdot 10^{-17}\,\text{J} \\ \Delta E &= E(3,3,3) - E(2,2,2) = \varepsilon_0\left((3^2+3^2+3^2) - (2^2+2^2+2^2)\right) \\ &= \varepsilon_0\left((27) - (12)\right) = 15\varepsilon_0 = 5E_{1,1,1} = \boxed{5.0 \cdot 10^{-17}\,\text{J.}} \end{aligned}$$

   (c) If a hydrogen atom begins in the $n = 4$ state, what is the value of the quantum number n of the final state in the transition that *emits the highest frequency* photon? **Solution:** The highest frequency emission would be for the greatest energy difference between the initial and final states. Since a photon is emitted in this case, the final state is at lower n than the initial state, and the lowest energy state is $\boxed{n = 1.}$

   (d) A particle is placed in a box of length a and has an uncertainty in momentum $\delta p = 1.0 \cdot 10^{-24}$ in SI units.

      i. What are the SI units for δp? **Solution:** Momentum is mass times velocity, so the SI units are $\boxed{\text{kg m s}^{-1}.}$

      ii. We then divide the length of the box in half. What does the uncertainty principle predict is the new value of δp? **Solution:** We've reduced the uncertainty in position by a factor of two, so δp must increase by a factor of at least two: $\delta p = \boxed{2.0 \cdot 10^{-24}\,\text{kg m s}^{-1}.}$

2. (a) Normalize the function

$$\psi(x) = \begin{cases} A(1+x) & 0 \le x < 3 \\ 0 & x < 0 \text{ and } 3 \le x \end{cases}$$

   Solution:

$$\begin{aligned} 1 &= \int_{\text{all space}} \psi(x)^2\,dx = \int_0^3 A^2(1+x)^2\,dx \\ &= A^2 \int_0^3 (1 + 2x + x^2)\,dx = A^2\left(x + x^2 + \frac{x^3}{3}\right)\Big|_0^3 \\ &= A^2\left[\left(3 + 9 + \frac{27}{3}\right) - 0\right] = 21\,A^2 \\ A &= \boxed{\sqrt{\frac{1}{21}}.} \end{aligned}$$

(b) Calculate the probability density of the particle between $x = 0$ and $x = 1$. **Solution:**

$$\int_0^1 \psi(x)^2\, dx = A^2 \left(x + x^2 + \frac{x^3}{3} \right)\Bigg|_0^1 = \frac{1}{21}\left(1 + 1 + \frac{1}{3}\right) = \frac{1}{21}\left(\frac{7}{3}\right) = \boxed{\frac{1}{9}.}$$

3. For each of the following particle-in-a-1D-box systems, write **A** or **B** to indicate which case has the *greater number of nodes* in the wavefunction, or write **both** if A and B have an equal number of nodes: **Solution:** The number of nodes equals $n - 1$ and depends only on the n value of the wavefunction.

   (a) (A) the $n = 2$ state of an electron
   (B) the $n = 4$ state of an electron in the same box.
   $\boxed{\text{B.}}$

   (b) (A) the $n = 2$ state of an electron
   (B) a proton at the same energy in the same box.
   $\boxed{\text{B.}}$ At the same energy, the more massive proton must be in a state with higher n.

   (c) (A) the $n = 2$ state of an electron
   (B) the $n = 2$ state of a proton in the same box.
   $\boxed{\text{Both.}}$

   (d) (A) the $n = 2$ state of an electron in a box of length a
   (B) an electron at the same energy in a box of length $2a$
   $\boxed{\text{B.}}$ To be at the same energy, the electron in the longer box must be at greater n.

   (e) (A) the $n = 2$ state of an electron in a box of length a
   (B) the $n = 2$ state of an electron in a box of length $2a$
   $\boxed{\text{Both.}}$

   (f) (A) the $n = 2$ state of an electron in a box of length a
   (B) the $n = 8$ state of an electron in a box of length $2a$
   $\boxed{\text{B.}}$

4. Imagine that Bohr had decided not to quantize the angular motion of the electron, but the radial motion instead, letting the electron travel only along a straight line (let's call it the x axis) through the nucleus. To simplify the problem, we'll use a different potential energy function, shown below:

$$U(x) = c\,|x| - E_0.$$

   If we apply classical mechanics the way Bohr did, this potential energy predicts that the electron will oscillate back and forth between two points x_n and $-x_n$, where the speed at any *positive* value of x is

$$v(x) = \sqrt{-\frac{2c}{m}(x - x_n)}.$$

   (a) If the electron is again treated as a de Broglie wave, will this atom radiate electromagnetic radiation? **Solution:** $\boxed{\text{No.}}$ The electron will be distributed evenly about the nucleus, and so there will be no electric dipole moment and no radiation.

   (b) Assume that the energy must still satisfy the equation $E = -b/n^2$ and solve for x_n in terms of b, c, m, n, and E_0.

Solution: We only need to solve in the region where $x > 0$, since the energy must be the same when the electron is traveling in the $x < 0$ region as well. That lets us set $U = cx$:

$$E_n = K + U = \frac{mv^2}{2} + cx - E_0 = \frac{m}{2}\left[-\left(\frac{2c}{m}\right)(x - x_n)\right] + cx - E_0$$
$$= -cx + cx_n + cx - E_0 = cx_n - E_0 = -\frac{b}{n^2}$$
$$x_n = \frac{E_0}{c} - \frac{b}{cn^2}.$$