Chemistry 410A **Exam 1** **Fall 2008**

NAME:

Instructions:

1. Keep this exam closed until instructed to begin.
2. **Please write your name on this page but not on any other page.**
3. Please silence any noisy electronic devices you have.
4. Attached sheet(s) provide potentially useful constants and equations. You may detach these from the exam.
5. To receive full credit for your work, please
   (a) show all your work, using only the exam papers, including the back of this sheet if necessary;
   (b) specify the correct units, if any, for your final answers;
   (c) use an appropriate number of significant digits for final numerical answers;
   (d) **stop writing and close your exam immediately when time is called.**

Other notes:

- **The first page portion of the exam is worth 40 points.** Partial credit for these problems is not necessarily available.
- **Your 2 best scores of the 3 remaining problems will count towards the other 60 points.** Partial credit is available for these problems, so try each problem and do not erase any of your work.

1. **40 points.**

(a) How much time does it take a photon to travel 20.0 m through a vacuum?

(b) Circle the letter for any of the following changes which *decrease* the de Broglie wavelength of a hydrogen atom traveling in a one-dimensional box.

i. Replacing the hydrogen atom with a helium atom, keeping E the same.

ii. Doubling the speed of the atom.

iii. Decreasing the particle-in-a-box quantum number n by 1.

iv. Reducing the length of the container by half, keeping n the same.

(c) Calculate the energy in J of the $n = 2$ state of a proton in a one-dimensional box of length 4.0 Å.

(d) If d/dx operates on $f(x) = 2e^{-3x}$, what is the eigenvalue?

2. If the uncertainty in position of an electron is $\delta x = 1.0\,\text{Å}$ and its average speed is $3.0 \cdot 10^6\,\mathrm{m\,s^{-1}}$, find the electron's de Broglie wavelength and the *minimum uncertainty* in the de Broglie wavelength. The relationship between the uncertainties is given by $\delta p / \delta \lambda_{\mathrm{dB}} = |dp/d\lambda_{\mathrm{dB}}|$.

3. The photon that excites the $n = 1 \rightarrow 10$ in He^+ has the same energy necessary to excite the $n = 5 \rightarrow 6$ transition in what other one-electron ion?

4. Find an equation for the root mean square speed $\langle v^2 \rangle^{1/2}$ of a particle with mass m in a one-dimensional box of length a and quantum state n.

$$\text{Balmer:}\quad E_{\text{photon}} = (2.180\cdot 10^{-18}\,\text{J})\left(\frac{1}{\text{n}''^2} - \frac{1}{\text{n}'^2}\right)$$

$$\text{Bohr atom:}\quad r_n = \frac{4\pi\epsilon_0 n^2\hbar^2}{Zm_e e^2} = \frac{n^2}{Z}a_0 \qquad v_n = \frac{Ze^2}{4\pi\epsilon_0 n\hbar}$$

$$E_n = -\frac{Z^2 m_e e^4}{(4\pi\epsilon_0)^2 2n^2\hbar^2} = -\frac{Z^2}{2n^2}\,E_{\text{h}} \qquad L_n = m_e r_n v_n = n\hbar$$

$$\text{momentum operator:}\quad \hat{p}_x = \frac{\hbar}{i}\frac{d}{dx}$$

$$\text{kinetic energy operator:}\quad \hat{K} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$$

$$\text{particle in a 1-D box:}\quad \psi(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \qquad E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

Derivatives

dx^c	$= cx^{c-1}dx\ (c \neq 0)$	$d(cx)$	$= cdx$
$d\ln x$	$= \frac{1}{x}dx$	de^x	$= e^x dx$
$d\sin x$	$= \cos x dx$	$d\cos x$	$= -\sin x dx$
$d[f(x)+g(x)]$	$= d[f(x)] + d[g(x)]$	$d[f(x)g(x)]$	$= f(x)d[g(x)] + g(x)d[f(x)]$

Integrals

$\int x^n dx$	$= \frac{1}{n+1}x^{n+1} + C$	$\int a dx$	$= a(x + C)$
$\int \frac{1}{x}dx$	$= \ln x + C$	$\int e^x dx$	$= e^x + C$
$\int \ln x dx$	$= x\ln x - x + C$	$\int \frac{dx}{x(a+bx)}$	$= -\frac{1}{a}\ln\left(\frac{a+bx}{x}\right) + C$
$\int \sin x dx$	$= -\cos x + C$	$\int \cos x dx$	$= \sin x + C$
$\int \sin^2(ax)dx$	$= \frac{x}{2} - \frac{\sin(2ax)}{4a} + C$	$\int \cos^2(ax)dx$	$= \frac{x}{2} + \frac{\sin(2ax)}{4a} + C$
$\int [f(x)+g(x)]dx$	$= \int f(x)dx + \int g(x)dx$	$\int_a^b dx$	$= x\vert_a^b = b - a$
$\int_0^\infty x^n e^{-ax}dx$	$= \frac{n!}{a^{n+1}}$	$\int_0^\infty e^{-ax^2}dx$	$= \frac{1}{2}\left(\frac{\pi}{a}\right)^{1/2}$
$\int_0^\infty xe^{-ax^2}dx$	$= \frac{1}{2a}$	$\int_0^\infty x^2 e^{-ax^2}dx$	$= \frac{1}{4}\left(\frac{\pi}{a^3}\right)^{1/2}$
$\int_0^\infty x^{2n+1}e^{-ax^2}dx$	$= \frac{n!}{2a^{n+1}}$	$\int_0^\infty x^{2n}e^{-ax^2}dx$	$= \frac{[1\cdot 3\cdot 5\ldots(2n-1)]\sqrt{\pi}}{2^{n+1}a^{n+(1/2)}}$

Fundamental Constants

Avogadro's number	$\mathcal{N}_A$	$6.0221367 \cdot 10^{23}$ mol^{-1}
Bohr radius	$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$	$5.29177249 \cdot 10^{-11}$ m
Boltzmann constant	k_B	$1.380658 \cdot 10^{-23}$ J K^{-1}
electron rest mass	m_e	$9.1093897 \cdot 10^{-31}$ kg
fundamental charge	e	$1.6021773 \cdot 10^{-19}$ C
permittivity factor	$4\pi\epsilon_0$	$1.113 \cdot 10^{-10}$ C^2 J^{-1} m^{-1}
gas constant	R	8.314510 J K^{-1} mol^{-1}
	R	0.08314510 L bar K^{-1} mol^{-1}
	R	0.08206 L atm K^{-1} mol^{-1}
hartree	$E_\mathrm{h} = \frac{m_e e^4}{(4\pi\epsilon_0)^2\hbar^2}$	$4.35980 \cdot 10^{-18}$ J
Planck's constant	h	$6.6260755 \cdot 10^{-34}$ J s
	$\hbar$	$1.05457266 \cdot 10^{-34}$ J s
proton rest mass	m_p	$1.6726231 \cdot 10^{-27}$ kg
neutron rest mass	m_n	$1.6749286 \cdot 10^{-27}$ kg
speed of light	c	$2.99792458 \cdot 10^{8}$ m s^{-1}

Unit Conversions

	K	cm^{-1}	kJ mol^{-1}	kcal mol^{-1}	erg	kJ
kHz =	$4.799 \cdot 10^{-8}$	$3.336 \cdot 10^{-8}$	$3.990 \cdot 10^{-10}$	$9.537 \cdot 10^{-11}$	$6.626 \cdot 10^{-24}$	$6.626 \cdot 10^{-34}$
MHz =	$4.799 \cdot 10^{-5}$	$3.336 \cdot 10^{-5}$	$3.990 \cdot 10^{-7}$	$9.537 \cdot 10^{-8}$	$6.626 \cdot 10^{-21}$	$6.626 \cdot 10^{-31}$
GHz =	$4.799 \cdot 10^{-2}$	$3.336 \cdot 10^{-2}$	$3.990 \cdot 10^{-4}$	$9.537 \cdot 10^{-5}$	$6.626 \cdot 10^{-18}$	$6.626 \cdot 10^{-28}$
K =	1	0.6950	$8.314 \cdot 10^{-3}$	$1.987 \cdot 10^{-3}$	$1.381 \cdot 10^{-16}$	$1.381 \cdot 10^{-26}$
cm^{-1} =	1.4388	1	$1.196 \cdot 10^{-2}$	$2.859 \cdot 10^{-3}$	$1.986 \cdot 10^{-16}$	$1.986 \cdot 10^{-26}$
kJ mol^{-1} =	$1.203 \cdot 10^{2}$	83.59	1	0.2390	$1.661 \cdot 10^{-14}$	$1.661 \cdot 10^{-24}$
kcal mol^{-1} =	$5.032 \cdot 10^{2}$	$3.498 \cdot 10^{2}$	4.184	1	$6.948 \cdot 10^{-14}$	$6.948 \cdot 10^{-24}$
eV =	$1.160 \cdot 10^{4}$	$8.066 \cdot 10^{3}$	96.49	23.06	$1.602 \cdot 10^{-12}$	$1.602 \cdot 10^{-22}$
hartree =	$3.158 \cdot 10^{5}$	$2.195 \cdot 10^{5}$	$2.625 \cdot 10^{3}$	$6.275 \cdot 10^{2}$	$4.360 \cdot 10^{-11}$	$4.360 \cdot 10^{-21}$
erg =	$7.243 \cdot 10^{15}$	$5.034 \cdot 10^{15}$	$6.022 \cdot 10^{13}$	$1.439 \cdot 10^{13}$	1	10^{-10}
J =	$7.243 \cdot 10^{22}$	$5.034 \cdot 10^{22}$	$6.022 \cdot 10^{20}$	$1.439 \cdot 10^{20}$	10^{7}	10^{-3}
dm^3 bar =	$7.243 \cdot 10^{24}$	$5.034 \cdot 10^{24}$	$6.022 \cdot 10^{22}$	$1.439 \cdot 10^{22}$	$1.000 \cdot 10^{9}$	0.1000
kJ =	$7.243 \cdot 10^{25}$	$5.034 \cdot 10^{25}$	$6.022 \cdot 10^{23}$	$1.439 \cdot 10^{23}$	10^{10}	1

distance	1 Å =	10^{-10} m	
mass	1 amu =	$1.66054 \cdot 10^{-27}$ kg	
energy	1 J =	1 kg m^2 s^{-2}	= 10^7 erg
force	1 N =	1 kg m s^{-2}	= 10^5 dyn
electrostatic charge	1 C =	1 A s	= $2.9979 \cdot 10^9$ esu
	1 D =	$3.3357 \cdot 10^{-30}$ C m	= $1 \cdot 10^{-18}$ esu cm
magnetic field strength	1 T =	1 kg s^{-2} A^{-1}	= 10^4 gauss
pressure	1 Pa =	1 N m^{-2}	= 1 kg m^{-1} s^{-2}
	1 bar =	10^5 Pa	= 0.98692 atm