## Solutions

## 1. 40 points.

(a) Write the complete MO configuration that you would expect for $\mathrm{B}_{2}$, if you follow the line for $\mathrm{N}_{2}$ in the schematic homonuclear diatomics correlation diagram (Fig. 6.2). Solution: $\mathrm{B}_{2}$ has 10 electrons, and we can put two to an orbital: $1 \sigma_{g}^{2} 1 \sigma_{u}^{2} 2 \sigma_{g}^{2} 2 \sigma_{u}^{2} 1 \pi_{u}^{2}$.
(b) Write the term symbol for the $\mathrm{BH}_{3}^{+}$ion, with MO configuration $1 a_{1}^{\prime 2} 2 a_{1}^{\prime 2} 1 e^{\prime 3}$. Solution: $S=1 / 2$ and the symmetry of $e^{\prime 3}$ is the same as $e^{\prime}$, so ${ }^{2} E^{\prime}$.
(c) Circle the molecule that you would expect to have the lowest vibrational constant $\omega_{e}$ : Solution: All three molecules are made from atoms with the same valence, so will have similar bonding. In that case, the more massive atoms will give the higher $\mu$ and lower $k$ :
MgO
CaO
CaS
(d) Circle the molecule that you would expect to have the lowest vibrational constant for the CC stretch: Solution: In this case, the reduced masses are similar but the $k$ values increase from single- to double- to triple-bond:
$\mathrm{H}_{3} \mathrm{CCH}_{3}$
$\mathrm{H}_{2} \mathrm{CCH}_{2}$
HCCH
(e) For the $v=3$ state of a simple harmonic oscillator:
i. Write the wavefunction in terms of the unitless coordinate $y$. Solution: From Table 7.1:

$$
\psi_{v=3}=A_{3} H_{3} e^{-y^{2} / 2}=\left(\frac{k \mu}{\hbar^{2}}\right)^{1 / 8}\left(\frac{1}{48 \sqrt{\pi}}\right)^{1 / 2}\left(8 y^{3}-12 y\right) e^{-y^{2} / 2}
$$

ii. How many nodes does this wavefunction have? Solution: 3 .
(f) How many vibrational modes are there in the simplest amino acid, glycine $\left(\mathrm{NH}_{2} \mathrm{CH}_{2} \mathrm{COOH}\right)$ ? Solution: Non-linear with $N=10$ atoms, so $3 N-6=24$.
2. Ground state acetylene is linear, but its lowest excited state is a triplet state with $C_{2 h}$ symmetry. Give the representations in the $C_{2 h}$ limit that correlate to the MO's listed below for the linear molecule.

$$
\mathrm{H}-\mathrm{C} \equiv \mathrm{C}-\mathrm{H}
$$



Solution: From the linear to the bent structure, the symmetry elements of $C_{2 h}$ are conserved. One of the $\hat{\sigma}_{v}$ planes in $D_{\infty h}$ becomes the $\hat{\sigma}_{h}$ plane in $C_{2 h}$, One of the $\hat{C}_{2}$ axes in $D_{\infty h}$ (perpendicular to the internuclear axis) becomes the principal $\hat{C}_{2}$ axis in $C_{2 h}$, and $\hat{I}$ is unchanged. Therefore, we can just keep track of the symmetry under $\hat{C}_{2}$ and $\hat{I}$ to find the correlating representation in $C_{2 h}$ :

| group | $\Gamma\left(D_{\infty h}\right)$ | $\hat{C}_{2}$ | $\hat{I}$ | $\Gamma\left(C_{2 h}\right)$ |
| :---: | :---: | ---: | ---: | :---: |
| $\mathrm{C}-\mathrm{H}$ | $\sigma_{g}$ | 1 | 1 | $a_{g}$ |
| $\mathrm{C}-\mathrm{H}$ | $\sigma_{u}$ | -1 | -1 | $b_{u}$ |
| $\mathrm{C} \equiv \mathrm{C}$ | $\sigma_{g}$ | 1 | 1 | $a_{g}$ |
| $\mathrm{C} \equiv \mathrm{C}$ | $\pi_{u}$ in - plane | -1 | -1 | $b_{u}$ |
|  | $\pi_{u}$ out - of - plane | 1 | -1 | $a_{u}$ |

3. Give the symmetry representation for each of the vibrational modes below, and indicate whether each mode is IR-active or IR-inactive (in other words, can the mode be excited by an allowed eletctric dipole transition).

a) $b_{u}$

IR-active

b) $a_{g}$

IR-inactive

c) $a_{u}$

IR-active

## Solution:

4. The $v=0 \rightarrow 1$ transition in $\mathrm{CH}^{+}$is measured at $2046.3 \mathrm{~cm}^{-1}$. If the force constant is $259.0 \mathrm{~N} \mathrm{~m}^{-1}$, calculate the anharmonicity $\omega_{e} x_{e}$. Solution: The transition energy depends on $\omega_{e}$ and $\omega_{e} x_{e}$. Therefore, to get $\omega_{e} x_{e}$ from the transition energy, we need to know the value of $\omega_{e}$. We can get $\omega_{e}$ from $\mu$ and $k$, so we're all set:

$$
\begin{aligned}
\mu & =\frac{(1.008)(12.00)}{1.008+12.00}=0.9299 \mathrm{amu} \\
\omega_{e}\left(\mathrm{~cm}^{-1}\right) & =130.28 \sqrt{\frac{k\left(\mathrm{Nm}^{-1}\right)}{\mu(\mathrm{amu})}}=130.28 \sqrt{\frac{259.0 \mathrm{~N} \mathrm{~m}^{-1}}{0.9299 \mathrm{amu}}}=2174.3 \mathrm{~cm}^{-1} \\
\Delta E & =\omega_{e}\left[\left(v^{\prime}+\frac{1}{2}\right)-\left(v^{\prime \prime}+\frac{1}{2}\right)\right]-\omega_{e} x_{e}\left[\left(v^{\prime}+\frac{1}{2}\right)^{2}-\left(v^{\prime \prime}+\frac{1}{2}\right)^{2}\right] \\
& =\omega_{e}[(3 / 2)-(1 / 2)]-\omega_{e} x_{e}[(9 / 4)-(1 / 4)]=\omega_{e}-2 \omega_{e} x_{e}=2046.3 \mathrm{~cm}^{-1} \\
\omega_{e} x_{e} & =\frac{\omega_{e}-\Delta E}{2}=\frac{\left(2174.3 \mathrm{~cm}^{-1}\right)-\left(2046.3 \mathrm{~cm}^{-1}\right)}{2}=64.0 \mathrm{~cm}^{-1} .
\end{aligned}
$$

