## Exam 5 Solutions

## 1. 40 points.

- (a) Write the complete MO configuration that you would expect for  $B_2$ , if you follow the line for  $N_2$  in the schematic homonuclear diatomics correlation diagram (Fig. 6.2). **Solution:**  $B_2$  has 10 electrons, and we can put two to an orbital:  $1\sigma_g^2 1\sigma_u^2 2\sigma_g^2 2\sigma_u^2 1\pi_u^2$ .
- (b) Write the term symbol for the BH<sub>3</sub><sup>+</sup> ion, with MO configuration  $1a'_{1}{}^{2}2a'_{1}{}^{2}1e'^{3}$ . Solution: S = 1/2 and the symmetry of  $e'^{3}$  is the same as e', so 2E'.
- (c) Circle the molecule that you would expect to have the **lowest** vibrational constant  $\omega_e$ : **Solution:** All three molecules are made from atoms with the same valence, so will have similar bonding. In that case, the more massive atoms will give the higher  $\mu$  and lower k: MgO CaO CaS
- (d) Circle the molecule that you would expect to have the **lowest** vibrational constant for the CC stretch: **Solution:** In this case, the reduced masses are similar but the k values increase from single- to double- to triple-bond:  $H_3CCH_3$   $H_2CCH_2$  HCCH
- (e) For the v = 3 state of a simple harmonic oscillator:
  - i. Write the wavefunction in terms of the unitless coordinate y. Solution: From Table 7.1:

$$\psi_{v=3} = A_3 H_3 e^{-y^2/2} = \left(\frac{k\mu}{\hbar^2}\right)^{1/8} \left(\frac{1}{48\sqrt{\pi}}\right)^{1/2} (8y^3 - 12y) e^{-y^2/2}.$$

ii. How many nodes does this wavefunction have? Solution: 3

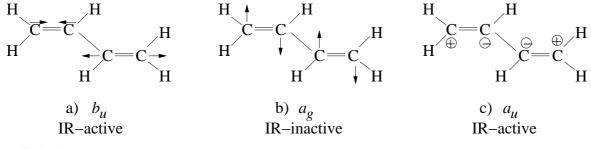
- (f) How many vibrational modes are there in the simplest amino acid, glycine  $(NH_2CH_2COOH)$ ? Solution: Non-linear with N = 10 atoms, so 3N 6 = 24.
- 2. Ground state acetylene is linear, but its lowest excited state is a triplet state with  $C_{2h}$  symmetry. Give the representations in the  $C_{2h}$  limit that correlate to the MO's listed below for the linear molecule.

$$H - C \equiv C - H$$
  $H : C = C :$ 

**Solution:** From the linear to the bent structure, the symmetry elements of  $C_{2h}$  are conserved. One of the  $\hat{\sigma}_v$  planes in  $D_{\infty h}$  becomes the  $\hat{\sigma}_h$  plane in  $C_{2h}$ , One of the  $\hat{C}_2$  axes in  $D_{\infty h}$  (perpendicular to the internuclear axis) becomes the principal  $\hat{C}_2$  axis in  $C_{2h}$ , and  $\hat{I}$  is unchanged. Therefore, we can just keep track of the symmetry under  $\hat{C}_2$  and  $\hat{I}$  to find the correlating representation in  $C_{2h}$ :

group	$\Gamma(D_{\infty h})$	$\hat{C}_2$	$\hat{I}$	$\Gamma(C_{2h})$
C-H	$\sigma_{g}$	1	1	$a_g$
С—Н	$\sigma_u$	-1	-1	$b_u$
C≡C	$\sigma_{g}$	1	1	$a_g$
C≡C	$\pi_u$ in – plane	-1	-1	$b_u$
	$\pi_u$ out – of – plane	1	-1	$a_u$

3. Give the symmetry representation for each of the vibrational modes below, and indicate whether each mode is IR-active or IR-inactive (in other words, can the mode be excited by an allowed electric dipole transition).



Solution:

4. The  $v = 0 \rightarrow 1$  transition in CH<sup>+</sup> is measured at 2046.3 cm<sup>-1</sup>. If the force constant is 259.0 N m<sup>-1</sup>, calculate the anharmonicity  $\omega_e x_e$ . Solution: The transition energy depends on  $\omega_e$  and  $\omega_e x_e$ . Therefore, to get  $\omega_e x_e$  from the transition energy, we need to know the value of  $\omega_e$ . We can get  $\omega_e$  from  $\mu$  and k, so we're all set:

$$\mu = \frac{(1.008)(12.00)}{1.008 + 12.00} = 0.9299 \text{ amu}$$
  

$$\omega_e (\text{cm}^{-1}) = 130.28 \sqrt{\frac{k (\text{N} \text{m}^{-1})}{\mu (\text{amu})}} = 130.28 \sqrt{\frac{259.0 \text{ N} \text{m}^{-1}}{0.9299 \text{ amu}}} = 2174.3 \text{ cm}^{-1}$$
  

$$\Delta E = \omega_e \left[ (v' + \frac{1}{2}) - (v'' + \frac{1}{2}) \right] - \omega_e x_e \left[ (v' + \frac{1}{2})^2 - (v'' + \frac{1}{2})^2 \right]$$
  

$$= \omega_e \left[ (3/2) - (1/2) \right] - \omega_e x_e \left[ (9/4) - (1/4) \right] = \omega_e - 2\omega_e x_e = 2046.3 \text{ cm}^{-1}$$
  

$$\omega_e x_e = \frac{\omega_e - \Delta E}{2} = \frac{(2174.3 \text{ cm}^{-1}) - (2046.3 \text{ cm}^{-1})}{2} = \boxed{64.0 \text{ cm}^{-1}}.$$