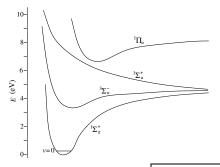
Exam 6 Solutions

1. 40 points.

(a) Given the set of potential energy curves drawn below, what is the likeliest process to be induced by radiation at a photon energy of 8.0 eV, starting from the v = 0 state shown? Identify any of the electronic states involved.



Solution: photodissociation from ${}^{1}\Sigma_{u}^{+}$. Transition to the ${}^{3}\Sigma_{u}^{-}$ is forbidden, so not likely. The transition to the ${}^{1}\Pi_{u}$ is allowed, but because the transition must occur vertically from v = 0, 8.0 eV is not enough energy to reach that state. The transition to the dissociating ${}^{1}\Sigma_{u}^{+}$ state is both allowed and within the energy range of the photon.

(b) The vibrational constant of ${}^{14}N_2$ in the ${}^{3}\Delta_u$ excited state is $1539 \,\mathrm{cm}^{-1}$. Find the force constant in SI units to three significant digits. Solution:

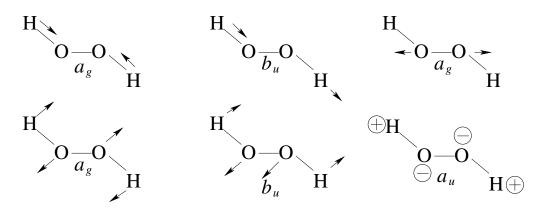
$$\omega_e \,(\,\mathrm{cm}^{-1}) = 130.28 \sqrt{\frac{k \,(\,\mathrm{N}\,\mathrm{m}^{-1})}{\mu \,(\,\mathrm{amu})}}$$
$$k = \left(\frac{\omega_e \,(\,\mathrm{cm}^{-1})}{130.28}\right)^2 \,\mu \,(\,\mathrm{amu}) = \left(\frac{1539}{130.28}\right)^2 \,(7.000) = \boxed{977 \,\mathrm{N}\,\mathrm{m}^{-1}}.$$

(c) Using the table of vibrational and rotational constants attached to the exam, calculate the total energy in rotation and vibration of the v = 0, J = 10 state of ${}^{12}C^{16}O$, including any corrections for which the data is available. Solution:

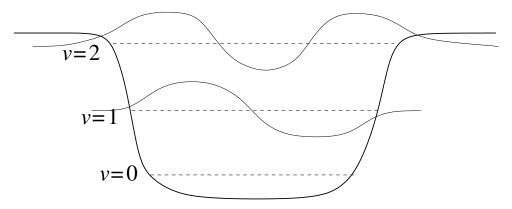
$$\begin{split} E_{\rm vib} &= \omega_e \left(v + \frac{1}{2} \right) - \omega_e x_e \left(v + \frac{1}{2} \right)^2 \\ &= (2169.82 \,{\rm cm}^{-1})(0.5) - (13.29 \,{\rm cm}^{-1})(0.5)^2 = 1081.59 \,{\rm cm}^{-1} \\ E_{\rm rot} &= [B_e - \alpha_e (v + \frac{1}{2})]J(J+1) - D_v [J(J+1)]^2 \\ &= [(1.9313 \,{\rm cm}^{-1}) - (0.0175 \,{\rm cm}^{-1})(0.5)](10)(11) - (6 \cdot 10^{-6} \,{\rm cm}^{-1})[(10)(11)]^2 = 211.41 \,{\rm cm}^{-1} \\ E_{\rm vib} + E_{\rm rot} &= (1081.59 + 211.41) \,{\rm cm}^{-1} = \boxed{1293.00 \,{\rm cm}^{-1}}. \end{split}$$

- 2. Hydrogen peroxide, HOOH, has the structure shown below.
 - (a) How many vibrational modes does HOOH have? Solution: $3N_{\text{atom}} 6 = 6$.
 - (b) Draw displacement arrows for each mode on the structures below.

(c) Label the representation for each mode.



3. The v = 0 state of a vibrational mode is shown on the potential energy curve below. Sketch as accurately as you can the energy levels and wavefunctions of the v = 1and v = 2 states.



4. We found a formula for the potential energy of interaction for two fixed dipoles that were co-aligned. Use the same approach to find the potential energy as a function of R and in terms of the dipole moments $\mu_{\rm A}$ and $\mu_{\rm B}$ for the case when the two dipole moments are parallel. **Solution:** The distance $q_{\rm A}$ to $-q_{\rm B}$ and $-q_{\rm A}$ to $q_{\rm B}$ is R. Set R' equal to the distance $q_{\rm A}$ to $q_{\rm B}$ and $-q_{\rm A}$ to $-q_{\rm B}$. Then $R' = \sqrt{R^2 + d^2}$. The rest opf the analysis follows the same strategy as for obtaining the dipole-dipole interaction potential:

$$\begin{split} u_{2-2}(R) &= \frac{1}{4\pi\epsilon_0} \left[\frac{(-q_{\rm A})q_{\rm B}}{R} + \frac{q_{\rm B}(-q_{\rm A})}{R} + \frac{q_{\rm A}q_{\rm B}}{\sqrt{R^2 + d^2}} + \frac{(-q_{\rm A})(-q_{\rm B})}{\sqrt{R^2 + d^2}} \right] \\ &= \frac{2q_{\rm A}q_{\rm B}}{4\pi\epsilon_0} \left[-\frac{2}{R} + \frac{2}{\sqrt{R^2 + d^2}} \right] = -\frac{2q_{\rm A}q_{\rm B}}{4\pi\epsilon_0 R} \left[1 - \left(1 + \frac{d^2}{R^2} \right)^{-1/2} \right] \\ &\approx -\frac{2q_{\rm A}q_{\rm B}}{4\pi\epsilon_0 R} \left[1 - \left(1 - \frac{d^2}{2R^2} \right) \right] \\ &= -\frac{2q_{\rm A}q_{\rm B}}{4\pi\epsilon_0 R} \left[\frac{d^2}{2R^2} \right] = -\frac{(q_{\rm A}d)(q_{\rm B}d)}{4\pi\epsilon_0 R^3} = -\frac{\mu_{\rm A}\mu_{\rm B}}{4\pi\epsilon_0 R^3} \end{split}$$