Exam 6
Solutions

## 1. 40 points.

(a) Given the set of potential energy curves drawn below, what is the likeliest process to be induced by radiation at a photon energy of 8.0 eV , starting from the $v=0$ state shown? Identify any of the electronic states involved.


Solution: photodissociation from ${ }^{1} \Sigma_{u}^{+}$. Transition to the ${ }^{3} \Sigma_{u}^{-}$is forbidden, so not likely. The transition to the ${ }^{1} \Pi_{u}$ is allowed, but because the transition must occur vertically from $v=0,8.0 \mathrm{eV}$ is not enough energy to reach that state. The transition to the dissociating ${ }^{1} \Sigma_{u}^{+}$state is both allowed and within the energy range of the photon.
(b) The vibrational constant of ${ }^{14} \mathrm{~N}_{2}$ in the ${ }^{3} \Delta_{u}$ excited state is $1539 \mathrm{~cm}^{-1}$. Find the force constant in SI units to three significant digits. Solution:

$$
\begin{aligned}
\omega_{e}\left(\mathrm{~cm}^{-1}\right) & =130.28 \sqrt{\frac{k\left(\mathrm{Nm}^{-1}\right)}{\mu(\mathrm{amu})}} \\
k & =\left(\frac{\omega_{e}\left(\mathrm{~cm}^{-1}\right)}{130.28}\right)^{2} \mu(\mathrm{amu})=\left(\frac{1539}{130.28}\right)^{2}(7.000)=977 \mathrm{Nm}^{-1} .
\end{aligned}
$$

(c) Using the table of vibrational and rotational constants attached to the exam, calculate the total energy in rotation and vibration of the $v=0, J=10$ state of ${ }^{12} \mathrm{C}^{16} \mathrm{O}$, including any corrections for which the data is available. Solution:

$$
\begin{aligned}
E_{\mathrm{vib}} & =\omega_{e}\left(v+\frac{1}{2}\right)-\omega_{e} x_{e}\left(v+\frac{1}{2}\right)^{2} \\
& =\left(2169.82 \mathrm{~cm}^{-1}\right)(0.5)-\left(13.29 \mathrm{~cm}^{-1}\right)(0.5)^{2}=1081.59 \mathrm{~cm}^{-1} \\
E_{\mathrm{rot}} & =\left[B_{e}-\alpha_{e}\left(v+\frac{1}{2}\right)\right] J(J+1)-D_{v}[J(J+1)]^{2} \\
& =\left[\left(1.9313 \mathrm{~cm}^{-1}\right)-\left(0.0175 \mathrm{~cm}^{-1}\right)(0.5)\right](10)(11)-\left(6 \cdot 10^{-6} \mathrm{~cm}^{-1}\right)[(10)(11)]^{2}=211.41 \mathrm{~cm}^{-1} \\
E_{\mathrm{vib}}+E_{\mathrm{rot}} & =(1081.59+211.41) \mathrm{cm}^{-1}=1293.00 \mathrm{~cm}^{-1} .
\end{aligned}
$$

2. Hydrogen peroxide, HOOH , has the structure shown below.
(a) How many vibrational modes does HOOH have? Solution: $3 N_{\text {atom }}-6=6$.
(b) Draw displacement arrows for each mode on the structures below.
(c) Label the representation for each mode.


3. The $v=0$ state of a vibrational mode is shown on the potential energy curve below. Sketch as accurately as you can the energy levels and wavefunctions of the $v=1$ and $v=2$ states.

4. We found a formula for the potential energy of interaction for two fixed dipoles that were co-aligned. Use the same approach to find the potential energy as a function of $R$ and in terms of the dipole moments $\mu_{\mathrm{A}}$ and $\mu_{\mathrm{B}}$ for the case when the two dipole moments are parallel. Solution: The distance $q_{\mathrm{A}}$ to $-q_{\mathrm{B}}$ and $-q_{\mathrm{A}}$ to $q_{\mathrm{B}}$ is $R$. Set $R^{\prime}$ equal to the distance $q_{\mathrm{A}}$ to $q_{\mathrm{B}}$ and $-q_{\mathrm{A}}$ to $-q_{\mathrm{B}}$. Then $R^{\prime}=\sqrt{R^{2}+d^{2}}$. The rest opf the analysis follows the same strategy as for obtaining the dipole-dipole interaction potential:

$$
\begin{aligned}
u_{2-2}(R) & =\frac{1}{4 \pi \epsilon_{0}}\left[\frac{\left(-q_{\mathrm{A}}\right) q_{\mathrm{B}}}{R}+\frac{q_{\mathrm{B}}\left(-q_{\mathrm{A}}\right)}{R}+\frac{q_{\mathrm{A}} q_{\mathrm{B}}}{\sqrt{R^{2}+d^{2}}}+\frac{\left(-q_{\mathrm{A}}\right)\left(-q_{\mathrm{B}}\right)}{\sqrt{R^{2}+d^{2}}}\right] \\
& =\frac{2 q_{\mathrm{A}} q_{\mathrm{B}}}{4 \pi \epsilon_{0}}\left[-\frac{2}{R}+\frac{2}{\sqrt{R^{2}+d^{2}}}\right]=-\frac{2 q_{\mathrm{A}} q_{\mathrm{B}}}{4 \pi \epsilon_{0} R}\left[1-\left(1+\frac{d^{2}}{R^{2}}\right)^{-1 / 2}\right] \\
& \approx-\frac{2 q_{\mathrm{A}} q_{\mathrm{B}}}{4 \pi \epsilon_{0} R}\left[1-\left(1-\frac{d^{2}}{2 R^{2}}\right)\right] \\
& =-\frac{2 q_{\mathrm{A}} q_{\mathrm{B}}}{4 \pi \epsilon_{0} R}\left[\frac{d^{2}}{2 R^{2}}\right]=-\frac{\left(q_{\mathrm{A}} d\right)\left(q_{\mathrm{B}} d\right)}{4 \pi \epsilon_{0} R^{3}}=-\frac{\mu_{\mathrm{A}} \mu_{\mathrm{B}}}{4 \pi \epsilon_{0} R^{3}}
\end{aligned}
$$

